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ARTICLE

A Mathematical Methodology for the Preventive Study of the Failure Rate to Optimize the Maintenance Program of a Public Work: Economic-Management Aspects for Safety and Quality

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ABSTRACT

In this paper we analyze the problem of the assessment of the failure rate of the complex public work system and the engineering part of it (bridge, tunnel, etc.), examining the case of serious maintenance problems, such as those which occurred in the recent disaster of the "Morandi bridge".

The original mathematical methodology envisaged makes it possible to optimize the safety and quality scenarios of the operation and infrastructure in question, also from an economic-management point of view, evaluating every aspect in an integrated way and for the entire period life.

The scientific results obtained are of particular interest for the study of maximization of the planning protocols of "terotechnological" interventions, providing a contribution to the science of programmed maintenance for the mobility networks and for more complex parts such as bridges and tunnels.

1. Introduction

The maintenance of the public work and above all of network infrastructures has become a conservation and the real science of the collective assets[1-7], those currently investing the complex sector of the scientific research, called in technical literature with the neologism of Terotechnology.

From a technical, economic and environmental point of view, this new cultural concept of maintaining design standards and improving the performance of the structure for the entire period life, answers key questions: where, how, when to intervene, with what technological measures

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and at what costs. The need to perform maintenance stems from two primary sources, but not always convergent: the safety and quality of the service offered and the economic efficiency on the other hand.

The latter, however, is not always antithetical compared to the first, in consideration of the fact that a delay in the maintenance intervention always involves costs far more relevant than those due to timely action (even better if taken care of before the situation worsens). This is shown, for example, by the simple analysis of the decaying curves of the com-positional materials of engineering work, as in the following figure, where the representative variables of the "road-paving" and those related to the planned maintenance according to a life-cycle cost analysis criterion. In this sense, terotechnology has an even greater impact on the maintenance of civil structures, landscape assets, and monuments, as well for the "time" and interest required and the effect it will have on the community. In particular, the maintenance of today concludes, in the modern econometric analysis of the management of complex work[8-12], also in order to maximize the safety and quality of the exercise, the role of a real "conservation strategy" of the public interest, is becoming an increasingly important aspect with the increase and the duration of engineering systems.

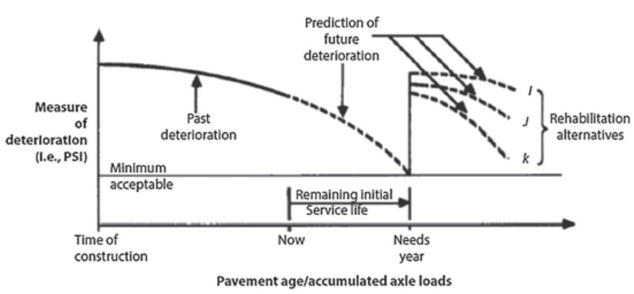

Figure 1. Pavement age/accumulated axle loads

Unfortunately, now we are inundated with news concerning natural disasters, accidents due to non-compliance with regulations, the lack of adequate maintenance programs and interventions to effectively prevent the risk of collapses, errors attributable to inadequate maintenance actions, or the lack of effective monitoring of the complex functioning system considered.

In the recent disaster in Genova, August 14, 2018 due to a serious deficiency of maintenance needed for the Morandi bridge in the Polcevera viaduct (A10 Genoa-Savona motorway), whose concession was given to the "Autostrade per l'Italia" company. The collapse of a bridge only adds a series of errors, design, maintenance and those whom authorized the transit of heavy vehicles, has not provided limitations on traffic and / or speed, narrowing of the roadway etc. For this strategic work, placed not only on the Liguria mobility network, but also on important transnational corridors, the Società Autostrade had already published a call for tenders for a substantial structural retrofitting operation.

But, on the subject of the "anomalies" of maintenance, other striking events have occurred that have shocked the international public in the past, such as that of June 3, 1998, near Eschede in Germany, when the worst high-speed train accident occurred: 101 people died and another 100 were seriously injured. The first cause of the accident was the breaking of a wheel rim in the first carriage, a part of the wheel on third axle of the convoy. Originally the wheel was cast in one piece but, subsequently, to reduce the vibrations that penalized the passenger's comfort, that was wrongly considered an "improvement" intervention, with the introduction of a special rubber ring between the wheel and rim: this modification was not compatible with the high speed of the train and was the primary cause of the disaster. The consequences of a deficient or inadequate maintenance is always and, in any case, very onerous not only socially but also economically and sometimes generate the loss of many human lives[13-19]. Sometimes the lack of "reliability" of work done can be fatal; however, the concept of reliability lies on the borderline between design and maintenance, while we now want to focus on the role of maintenance in the conservation of public work, until the maintenance and / or repair or reinforcement (in the case, of a crossing art work, such as the Morandi bridge, a varied configuration of traffic conditions over time and in particular the "flow-velocity" relationship), are not those of the project, requiring an accurate risk analysis, having changed with respect to the original forecasts. It should be noted, in this regard, that in its general meaning, the building process, as defined by the UNI 10838 standard, is the organized sequence of steps that lead from the detection of the needs of the client-user of a building property to their satisfaction through the design, production, construction and management of the asset itself.

2. The Variables Characterizing the "Maintenance-safety" Combination.

Therefore, an appropriate management program for the purposes of optimal conservation of the public health and safety, must include all the operational steps starting from the get go of the building body, are succeeded with the purpose of ensuring its proper functioning, until the end of its functional and economic life cycle[6,16,30,34]. As seen in Fig. 2 it shows the qualitative trends of the main cost functions and the overall degree of reliability of the engineering work.

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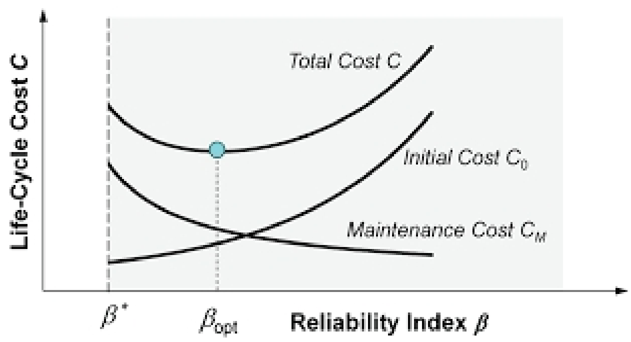

Figure 2. Performance of the main cost functions and the degree of reliability

The most recent definition of maintenance is that in the European standard UNI EN 13306 of 2003, where maintenance is described as the combination of all technical, administrative and management actions, during the life cycle of an entity, designed to keep it in good working order or bring it back to a state where it can perform the required function, in a safe condition.

The central mission of maintenance is therefore that of cooperating throughout the useful life cycle of a public work, from concept to disposal, with the aim to continuously improving its operational availability and maintaining the standard of reliability in operation to protect the public that gave rise to the investment. However, so that the concrete result in favor of the community is consistent with this "mission", the competent subjects must operatively guarantee, together with a steady monitoring of the work, the rigorous implementation of the system of actions defined for scheduled maintenance, in the envistaged thunderstorm.

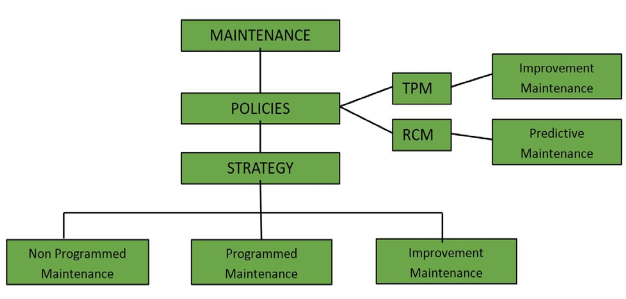

Figure 3. Scheme of the maintenance policy scenarios of a public work

3. A Mathematical Model for the Choice of the Solution that Maximizes the Objectives: the Case of Choice Under Conditions of Uncertainty

The most common methodological approaches to implement the planned maintenance scenarios of complex public work are essentially two: Total Productive Maintenance (TPM) and Reliability Centered Maintenance (RCM). The first leads to the Overall Efficiency, i.e. a more efficient use of plants and equipment that introduces a widespread maintenance methodology throughout the organization (Companywide) based on preventive maintenance - predictive (maintenance based on statistical data, as in the case of advanced "Railway terotechnology" implemented by the Italian Railway Network with diagnostic trains also for the AV / AC). In this way it is possible to intervene punctually, effectively and at the lowest operating cost, even before the deterioration occurs.

The RCM maintenance approach is centered on the concept of reliability, born in the aeronautical-military field, with the aim of consolidating the intrinsic reliability of the project. This strategy of optimizing the safety of the work uses, in fact, as a basis the reliability theory, that is a model of analysis of the causes of failure and the risk of the event. These elements allow the constructor responsible for the management and planning of the interventions to define the plans and the operational management methods, putting in place the actions of an integrated type on the complex of the compositional elements at work. In any case, whatever the model of reference, maintenance today requires a complex organization and an effective management system, oriented to the final and above all to the "prevention" of the fault; this implies, respect to the past, also a considerable cultural change of the technical management and of the same maintenance technician (or group of maintenance workers) who must carry out the work, often highly specialized and with the need for special technological equipment[20-24].

The choice of the best maintenance strategies, and consequently of the "conveniences" and of the opportunities for intervention, are different according to the case in question and, in time, in proportion to the transformation of the structure[25-28]. Furthermore, the type of choices and the actions required are mainly influenced by the trend of the failure rate [$\lambda(t)$] of a single component work and their structural interactions, as well as by particular exogenous events that occur (seismic actions, external corrosive agents, other factors of deterioration, etc.).

Unfortunately, in Italy, most of the maintenance interventions, as shown in the graph below, follow a failure strategy; exceptions are that the "predictive maintenance" programs carried out by the Railways, with the train measures that run through the entire network, measuring the characteristic parameters of the infrastructure. These measures are associated with the position: geographic coordinates measured by GPS with differential technology and path space. The position is subsequently linked to the

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progressive mileage and to the code of the technical headquarters of the infrastructure data base, which univocally identify the route or location in the company's asset management system.

Figure 4. Morandi Bridge (A10) - visible crack in the stack n. 9 (photo taken 7 minutes before the collapse – Fonte: huffingtonpost).

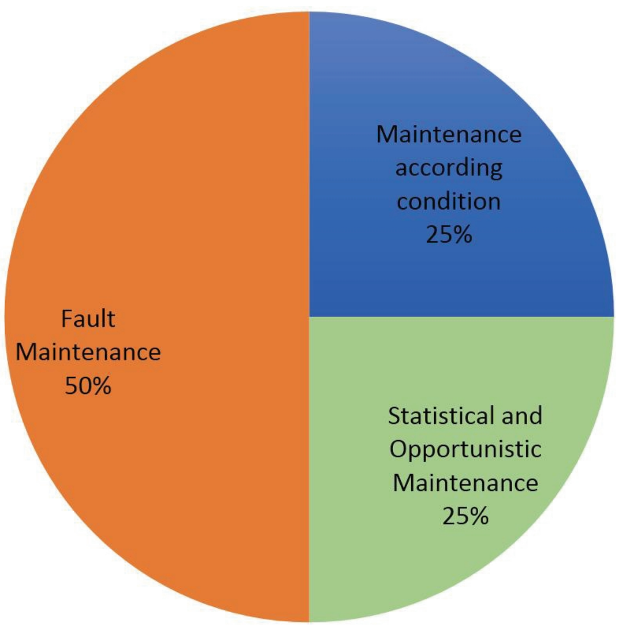

Figure 5. Maintenance culture in Italy

This indicates that the maintenance culture in the general management of public works respect to failure prevention is the tool to lengthen the period of life of the components that are not yet well structured in each sector, to obtain the maximum safety and quality and the minimization of costs over the entire useful life cycle (every predictive intervention, i.e. before the failure to verify, reduces the maintenance charges).

This deficiency emerges above all in the building sector where, in 64% of cases, corrective and non-preventive strategies are used. On the other hand, it is less deficient as regards the plant engineering fields, where the percentages of preventive measures are much higher than those with a breakdown (with the exception of electrical systems for public works and motorway tunnels). For the road transport sector, without prejudice to the large TEN-T motorway network, it should be noted that maintenance occurs most of the time, including for works of art such as bridges and viaducts, with which the road networks are richly equipped, in "emergency run-up" policy, which can be summarized according to the old "worst-first" culture (heal the worst situation first).

This illogical and uneconomical way of proceeding causes a significant increase in the level of risk, due to the ever-increasing volumes of traffic and therefore to the more critical characteristics of the flow that characterize the network conditions, jeopardizing the safety.

The companies responsible for managing the road patrimony in operation often have budget reductions made to their financial resources dedicated to road maintenance. Limiting even the highly specialized technical resources to comply with a continuous monitoring of the "state of health" of the works. In some cases, there is a lack of proper planning and the "rain" distribution of the overall budget provided on the network under concession, regardless of an accurate and timely assessment of priorities: so, the priority objective to guarantee the essential quality standards of gear and safety, not only to motorized users but to the entire population is nullified (as well as the tragic collapse of the Morandi bridge).

In particular, precisely for the work of crossing the territory, such as bridges, the activity inspection must be technically considered a systematic action that is framed over time for the entire useful life span, related to the continuous data collection of the adequate technological systems.

The analysis work will have to consider the historical series of observed variables, following the evolution of the overall performance and more so the safety level indicators, such as, to characterize the state of the bridge, the Condition Rating mark (CRM)).

Furthermore, in the case of the bridges made with concrete elements, a methodological risk an analysis process must be followed, functional to the systematic maintenance management, according to the diagram shown in Fig. 6.

The characteristic CRM indicator represents a numerical evaluation of all possible types of damage that have been detected during the visual inspection of the work and considers their gravity and extent. The condition of a bridge structure is evaluated by using the following general formula:

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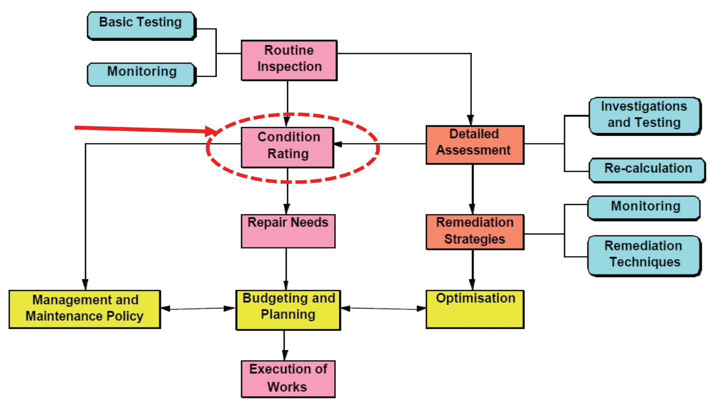

Figure 6. Risk analysis process for a concrete work of art

$$\boldsymbol{Condition\ Rating\ Mark} = \sum \boldsymbol{G_i \cdot K_{1i} \cdot K_{2i} \cdot K_{3i} \cdot K_{4i}}$$

where:

-G_i is the type of damage, $1 \leq G_i \leq 5$;

-K_{1i} is the extent of damage; $1 \leq K_{1i} \leq 1$. It can be described by the words: few or some, frequent or very frequent and big dimensions.

-K_{2i} is the extent of damage; $1 \leq K_{1i} \leq 1$. It can be described by the words: little or insignificant, medium, heavy and very heavy.

-K_{3i} is the importance of the structural element. $1 \leq K_{3i} \leq 1$. Structural components are classified as primary, secondary and other parts.

-K_{4i} is the urgency of intervention $1 \leq K_{4i} \leq 10$ and depends on the type of severity and risk of collapse of the structure or part of it.

-The values that can take Condition Rating Mark are between 0 and 70.

According to this approach, the degree of risk can therefore be roughly characterized by 6 different qualitative scenarios, as shown in the following table:

On the other hand, from a mathematical point of view, the fault alert levels can be identified by specific risk and criticality indicators of the event (detected or potential anomaly), from which the intervention priority can be obtained. Particularly useful for practical applications in the terotecnological sciences, they cover the so-called "risk matrices", as shown below.

Table 1. Degree of risk

Damage class	Definition	Condition Rating value
1	No or very little deterioration	0-3
2	Little deterioration	2-8
3	Medium to severe deterioration	6-13
4	Severe deterioration	10-25
5	Very severe deterioration	20-70
6	Very severe or total deterioration	>50

The latter is attributed on the basis of the variable RPN (Risk priority number) which is a composite risk index and denotes the degree of attention that is required in concrete to address the risk identified for each E_h component of the deterioration event E characterizing in its complex of the public work in the studio (bridge, gallery, entire network, etc.).

The systematic analysis that must be carried out must also include the evaluation of the degree of significance W_h ("weight") of each element E_h characterizing the maintenance problem examined, analyzing all the "concurrent causes" of the degradation.

Table 2. Risk Matrix

		Damage		
		Little	Moderate	Serious
Probability	Very Unlikely	Very Low Risk	Very Low Risk	High Risk
	Unlikely	Very Low Risk	Medium Risk	Very High Risk
	Likely	Low Risk	High Risk	Very High Risk
	Very Likely	Low Risk	Very High Risk	Very High Risk

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The "cause-effect" diagrams are the simplest and at the same time the most effective tool for dealing with the problem of the risk analysis of an event. The basis of this tool is to continually use the word "why", that is to constantly question the reason of a certain situation or phenomenon and proceed backwards by asking this question several times.

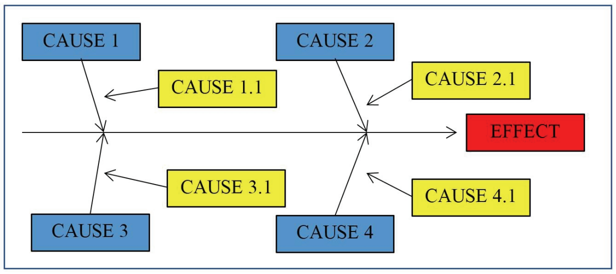

Figure 7. Cause-effect

In this way, it will be possible to make an operative reference to the "Failure Mode and Effect Analysis" criterion, systematically examining all the ways in which it is possible for damage to occur, and for each of them make an estimate of the effects through three indices that define the severity (S), the frequency of occurrence (f) and the detection capacity (C), evaluated according to a numerical scale and combined to define the aforementioned RPN index.

Following this approach, the Risk priority number can be calculated using the mathematical expression:

$$\boldsymbol{RPN}\,(\mathbf{E}) = \sum \boldsymbol{E_h}^{-\boldsymbol{W_h}} \left(\psi(x_i)\right) = \sum (\boldsymbol{S_h} \cdot \boldsymbol{f_h} \cdot \boldsymbol{C_h})^{-\boldsymbol{W_h}} \left(\psi(x_i)\right),$$

where $\sum w_h = 1.$

We must determine the values of x_1, x_2, .. x_k, in order to obtain the optimum solution of the objective function

$y = \psi\,(x_1^{-w1},..., x_1^{-wi} \,..\, x_h^{-wh}),$

where

$y = \psi\,(x_i^{-wi}),$

and $\sum w_i = 1$, and expresses the coordinated set of elements of optimization of safety and the quality of the operation of the work at the time "t" in which the verification is carried out, within the interval *[0, ..., t, ..., T]* of the relative useful life. The aim of the aforementioned analysis is to be able to identify in advance all the actions necessary for the full restoration of structural efficiency and performance in operation and to avoid all possible damage related to the risk event detected, ensuring the quality global engineering work and at the same time minimizing management costs, thanks to the timely adoption of the best solutions to the maintenance problems. Depending on the technical framework offered by the complex from the data collected and in relation to the risk scenario where decisions must be made in conditions of uncertainty (in the operational choices on maintenance the random variables often play a decisive role), a methodology approach in such case, the problem can be summarized as follows[29]. We consider m random independent events E_i (i = 1, …, m) which characterize the problem and we suppose that they are complementary, such that if the event E_1 occurs, it excludes any other alternative E_i.

For any other possible events we associate the probability p_i with $\sum_1^m p_i = 1.$ If we denote with A_j (i = 1,…,n) the different alternative options in oreder to make the maintenance program, the Project Manager must make his choices by analyzing a complex set I, made up of elements R_{ij} obtained from:

$A_j = f\,(E_i).$

The problem is represented by a data table, where the decision alternatives take on the mathematical connotation of random variables. Considering that the expected value of a discrete random variable (that assumes that is only a finite number or a countable infinity of values) is given by the sum of the possible values of this variable, each multiplied by the probability of being assumed (ie the occurrence of event), we need to calculate the weighted average of the possible results and, therefore, the expected value $V_j\,(A_j)$ of each variable A_j. The latter is given by the expression:

$$V_j\ (\ A_j\ ) = \sum_1^n R_{ij}\ \cdot p_i.$$

Therefore, on the basis of the previous reports, it will be possible to reach the "optimal" project solution $S_p(A_k)$, considering all the variables A_k that set in the definition of the decision-making process (including those A_j of random type):

$$\boldsymbol{S_p}\ (\ A_k\ ) = \boldsymbol{arg}\left\{\max_{\boldsymbol{k=1,\dots,s}}\ [V_k\ (\ A_k\ )^{-\boldsymbol{w_k}}]\right\},$$

where W_k represents the "weight" of each alternative A_k with $\sum_1^s w_k = 1.$.

In order to evaluate the degree of relevance and, therefore, make the best choice, the specific impact of each risk must be cataloged on the basis of a scale of factors such as performance, timing, costs, quality, the image of the organization, the damage to international relations, the effects to the environment, etc.

Finally, the probability of occurrence must be assigned to each identified risk and the expected monetary value estimated for each consequent impact generated (including the "externalities" due to the effects on the intrinsic safety of the work). This in order to be implement effectively the risk planning and management of better coordinated management of emergencies, providing for a targeted response

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strategy: this must ultimately include the definition of all the actions and resources necessary to reduce the impact and / or probability of risk within acceptable levels of safety and quality.

To this end, a logical scheme useful in practice to optimize this important aspect, can be represented by the following matrix (see Fig. 8) that provides a representation of the activities (A, ..., F) and the risk characterization, evaluated according to the criticality of the event and its probability of occurrence[30-33]

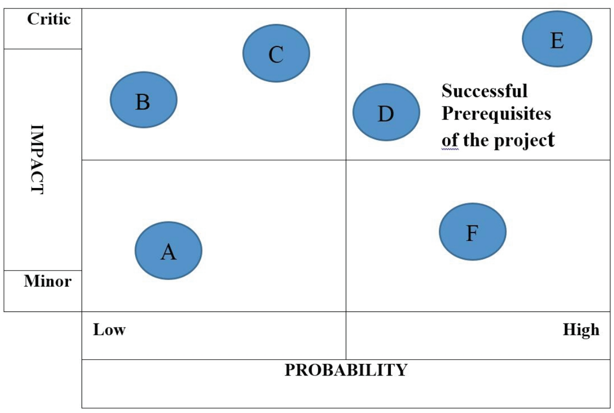

Figure 8. Logical scheme of probablity

Through this methodology of approach to the problem under study, we come to the choice of the "optimal" solution of the project $S_p(A_k)$, given by the formula:

$$\boldsymbol{S_p}\ (A_k) = \boldsymbol{arg}\left\{\max_{\boldsymbol{k=1,\dots,s}} [V_k\ (A_k)^{-w_k}]\right\},$$

where A_k is the generic project alternative of intervention, among the "s" compared, in respect to the different technical, economic and environmental constraints, while is the expected value of each variable A_k (many of them aleatory) analyzed in the decision-making process, to which a "weight" w_k has been attributed.

4. Preventive Failure Rate Study to Optimize Maintenance

Consider a sequence $X_1, X_2, \ldots$ of independent random variables of the same order, which represent periods of life[34]. It could either represent time of service before a failure of any material kind occurring at an initial time of t=0. Our aim is to obtain information on the common distribution of these periods of life starting from observation data. Putting $X=X_1$, for convenience' sake, we note that $F(x)=(X\leq x)$ is the probability that the length of life is less than or equal to x. It is supposed that F has a probability density $f(x) = \frac{d}{dx}F(x)$ continuous on $(0,\infty)$, which allows us to define the rate of failure as

$$\lambda(x) = \frac{f(x)}{1-F(x)} = \frac{\textit{Number of fail in the period}[x, x+dx]}{dx \times \textit{Mean number of observations } x} \tag{1}$$

The most classic model to present these periods of life is the exponential law $F(x) = 1 - \exp(-\lambda x)$ for $x > 0$, corresponding to a λ constant rate of failure. From a sample $X_1, X_2, \ldots X_n$ of data resulting from this exponential law, λ can be calculated without difficulty by the method of the maximum likelihood ratio

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^{n} X_i}, \tag{2}$$

which is asymptotically normal with $n^{1/2}(\hat{\lambda}/\lambda - 1) \to N(0,1)$ when $n \to \infty$. However, this is an ideal situation, and a more complicated distributions of exponential laws that are more frequent. The most classic example is given by the Weibull family of laws which includes the exponential law as a particular case, and for which,

$$F(x) = 1 - \exp(-\lambda x^{\alpha}),$$

and

$$f(x) = \alpha\lambda x^{\alpha-1}\exp(-\lambda x^{\alpha}) \qquad \textit{for } x > 0.$$

The corresponding rate of failure is therefore

$$\lambda(x) = \alpha\lambda x^{\alpha-1} \qquad \textit{for } x > 0\cdot$$

IFR (increasing failure rate) or DFR (decreasing failure rate) laws are thus obtained by putting a>1 or 0<a<1. Evaluation of λ and x parameters of such a distribution is more complex than the exponential law. This is in part due to the fact that density $f(x)$ can become infinite at x=0 for certain values of x. However, it is possible to use the method of the maximum likelihood ratio estimating (λ, α) for $(\hat{\lambda}, \hat{\alpha})$, obtained from the solution of the following equations

$$\hat{\lambda} = \frac{n}{\sum_{i=1}^{n} X_i^{\hat{\alpha}}} \qquad \textit{and} \qquad \frac{1}{\hat{\alpha}} = \frac{\sum_{i=1}^{n} X_i^{\hat{\alpha}} \log X_i}{\sum_{i=1}^{n} X_i^{\hat{\alpha}}} - \frac{\sum_{i=1}^{n} X_i^{\hat{\alpha}} \log X_i}{n} \quad .(3)$$

This is usually obtained by means of recurring algorithms, starting from the initial value $(\hat{\alpha}, \hat{\lambda})$of (α, λ), obtained from the moment's method. To obtain this, it can be sent that, for r>0 we have:

$$(X^r) = \Gamma(r/\alpha + 1)\lambda^{-r/\alpha},$$

$$(X) = \Gamma(1/\alpha + 1)\lambda^{-1/\alpha},$$

$$(X) = \Gamma(2/\alpha + 1)\lambda^{-2/\alpha},$$

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which gives values $(\hat{\alpha},\hat{\lambda})$, obtained from the solution of the system

$$\frac{\Gamma(2/\hat{\alpha}+1)}{\Gamma(1/\hat{\alpha}+1)^2}=\frac{\frac{1}{n}\sum_{i=1}^{n}X_i^2}{\left(\frac{1}{n}\sum_{i=1}^{n}X_i\right)^2} \quad and \quad \hat{\lambda}=\left(\frac{\Gamma(1/\hat{\alpha}+1)}{\frac{1}{n}\sum_{i=1}^{n}X_i}\right)^{\hat{\alpha}} \tag{4}$$

The complex, implicit form the equations (3) and (4) gives us an idea of the practical difficulties which can be encountered in the application of these methods based on non-exponential parametric models. In addition, it may happen that data has to be used which do not have a clear model of parameters. There-fore, different models are compared amongst themselves, for example, based on gamma and lognormal families of laws. More details on this can be found in the work of Kalbfleisch and Prentice[35].

5. A Mathematical Approach for a Problem of Preventive Maintenance

The problem becomes even trickier when, instead of looking at raw data for a period of life $X_1,X_2,...X_n$, there is censored data[36-42]. It is supposed that there exists a sequence $Y_1,Y_2,\ldots$ of the censored time whereby it is only possible to observe periods of life through the intermediary (Z_1, δ_1), (Z_2, δ_2), … where, for $i = 1,2, ...$,

$$Z_i=\min\{X_i,Y_i\} \quad and \quad \delta_i=\begin{cases}1 & when\ X_i\le Y_i\\ 0 & when\ X_i>Y_i\end{cases} \tag{5}$$

in other words Z_i, is the minimum value of length of life X_i and time of censored Y_i, while δi is equal to 1 if the length of life is not censored, otherwise it equals 0.

Actually, this is the most frequent situation. In industrial application situations one does not wait for material to fail (at the end of time X_i) to substitute it, in a preventive maintenance program (carried out at the end of time Y_i). Intervention can take place at the end of time Z_i=min$\{X_i, Y_i\}$ distinguishing cases δ_i=1 (intervention after failure) from δ_i=0 (intervention of preventive maintenance).

In practice, a model can be used where the sequence of lengths of life $\{X_i : i\ge 1\}$ and the successions of censored time $\{Y_i : i\ge 1\}$ are independent, and where each of these sequences are made up of independent and random variables of the same order, with

F(x)=($X \le x$), G(x)=($Y \le x$),

H(x)=($Z \le x$)=1-(1-F(x))(1-g(x)) (6)

for $x > 0$, when Y=Y_1 and Z=Z_1

The shape of the distribution of the censored data (that is of G) is, in general, totally unknown. What is still worse, is that there is no reason to suppose that this distribution function G is continuous. For example, in the case of preventive maintenance, censored times are often used, taking a finite value number, which corresponds to discrete laws. Let it be said we are not particularly interested in G, which is introduced as a negative parameter of modern urban life, here we try to eliminate it to recuperate data on F.

In the case of which $F(x)=1-\exp(-\lambda x)$ follows an exponential law, evaluation of the rate of failure λ (which here fully determines the distribution of the length of life) is relatively easy, even with censored data. The following reasoning is quite clear. Let us put

$$G\_(x)=\lim_{\varepsilon\downarrow 0}G(x-\varepsilon)\cdot$$

It can easily be seen that

$$H(x)=Z\,\square\, x)=H^{(0)}(x)+H^{(1)}(x),$$

when

$$H^{(0)}(x)=P(Z\le x \quad and \quad \delta=1)=\int_0^x(1-G\_(t))\,dF(t),$$

and

$$H^{(1)}(x)=P(Z\le x \quad and \quad \delta=0)=\int_0^\infty(1-F\_(t))\,dG(t),$$

$$p=(\delta=1)=(\delta)=\int_0^\infty(1-G\_(t))\,dF(t), \tag{7}$$

when δ=δ_1. From (7) it is implied that

$$p=(\delta)=\lambda\int_0^\infty(1-G\_(x))\exp(-\lambda x)dx=\lambda\int_0^\infty(1-G\_(x))\exp(-\lambda x)dx,$$

while

$$(Z)=\int_0^\infty x\,dH(x)=\int\ (1-H(x))dx=\int_0(1-G(x))\exp(-\lambda x)dx.$$

the estimator can be deduced

$$\hat{\lambda}=\frac{\sum_{i=1}^{n}\delta_i}{\sum_{i=1}^{n}Z_i}=\frac{n^{-1}\sum_{i=1}^{n}\delta_i}{n^{-1}\sum_{i=1}^{n}Z_i}=\frac{\overline{\delta}}{\overline{Z}}. \tag{8}$$

It can be seen that (8) takes us back to (2) in the case of non-censored data (since δ_1=1 for every i). This estimate has optimum properties (estimator of the maximum likelihood ratio). It can be shown without difficulty that

$$(Z^2)=2B=2\int_0^\infty x(1-G(x))\exp(-\lambda x)dx, \qquad (Z)=A=\int_0^\infty(1-G(x))\exp(-\lambda x)dx,$$

$$(Z\delta)=\lambda B=\lambda\int_0^\infty x(1-G(x))\exp(-\lambda x)dx \qquad (\delta)=\lambda A=\lambda\int_0^\infty(1-G(x))\exp(-\lambda x)dx,$$

$$Var(Z)=2B-A^2, \quad Cov(Z,\delta)=\lambda(B-A^2) \quad Var(\delta)=\lambda A(1-\lambda A)$$

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In addition, when $n \to \infty$,

$$n^{1/2}\left(\frac{\hat{\lambda}}{\lambda}-1\right)=n^{1/2}\left(\frac{\hat{\delta}}{XZ}-1\right)=n^{1/2}\left(\frac{1+n^{-1/2}\left\{n^{1/2}(\bar{\delta}-\lambda A)/\lambda A\right\}}{1+n^{-1/2}\left\{n^{1/2}(Z-A)/A\right\}}-1\right)$$

$$=n^{-1/2}(\bar{\delta}-\lambda A)/\lambda A-n^{1/2}(\bar{Z}-A)/A =$$

$$=N\left(0,\frac{1}{\lambda A}\right)=N\left(0,\frac{1}{\lambda\int_0^1(1-G(x))e^{-\lambda x}dx}\right). \tag{9}$$

In this case of non-censored data results are found putting $G(x)$=1, and we obtain

$$\lambda\int_0^1(1-G(x))e^{-\lambda x}dx=\lambda\int_0^1 e^{-\lambda x}dx=1\cdot$$

The convergence (9) allows us to obtain confidence intervals for λ if we consider

$$n^{1/2}\left(\frac{\hat{\lambda}}{\lambda}-1\right)\bar{\delta}^{1/2}\xrightarrow{d}N(0,1) \quad with \quad \bar{\delta}=\frac{1}{n}\sum_{i=1}^{n}\delta_i$$

if it is underlined that

$$n\bar{\delta}=\sum_{i=1}^{n}\delta_i=N$$

is nothing more that the number of observations of the non-censored length of life, the asymptotic confidence interval is obtained

$$\left(\lambda\in\left[\hat{\lambda}\left(1-\frac{v_{\alpha/2}}{\sqrt{N}}\right),\hat{\lambda}\left(1-\frac{v_{\alpha/2}}{\sqrt{N}}\right)\right]\right)\approx 1-\alpha,$$

where $v_{\alpha/2}$ indicates the order quantity $1-\alpha/2$ of the normal law $N(0, 1)$. The formula above the above formulas are generalised, not without some technical difficulties, for the Weibull, gamma or lognormal family of laws. However, in the absence of precise information on models it is strongly advised to use non-parametric estimators such as the Kaplan-Meier (1958) estimator.

6. Conclusion

The recent and dramatic collapse of the Morandi Bridge on the A10 motorway, in Genova, has opened a lively debate on the investments actually incurring in Italy and in the EU for the maintenance and modernization of both the road and motorway networks, and moreover, on the whole transport system and on major crossing works.

The technical literature of the sector shows how an effective risk prevention tool is already represented by an accurate Bridge Management System (see Fig. 9), through which the managing body can continuously have all the information necessary to program in an optimal manner the maintenance and control of the structures heritage, considering structural, economic and social factors.

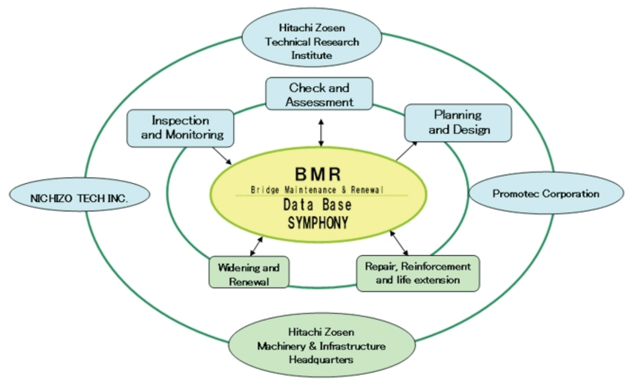

Figure 9. Bridge Management System

The OECD data show that Italy would rank second in Europe for road maintenance spending, with just over € 15,000 spent per km in the years between 2010 and 2015. However, these figures are not strictly indicative, because the Italian orographic context requires proportionately higher expenses than those that might be needed in another flat country of the Union such as, for example, Denmark.

Figure 10. Morandi Bridge collapsed in 14.8.2014 Genova (A10)

In the last decade, the science of maintenance (terotechnology) has had a significant development in the scientific field due to the incidence that it assumes in the economy of a modern country and for the community, representing a fundamental element for the exercise in safety of the complex integrated mobility system, both for passenger and freight traffic.

In the present work, attention has been focused on the assessment and analysis of the degradation status of the road infrastructures and in particular to the major works of art, in order to estimate the degree of risk of each compositional element and to establish the priorities of the interventions. maintenance necessary to ensure appropriate quality and safety standards.

Moreover, original mathematical models have been developed which, through the use of specific variables characterizing the problem under study, allow to achieve the optimization of the results of the planned maintenance action, both in terms of quality and safety and containment. Operating costs in the useful life cycle of the public works (or the set of assets, in the case of a network) under consideration.

The Project optimum solution (considering different aspect) $S_p(A_k)$ is taken from:

$$\boldsymbol{S_p}\ (A_k) = \boldsymbol{arg}\left\{\max_{\boldsymbol{k=1,\ldots,s}} \left[V_k\ (A_k)^{-w_k}\right]\right\},$$

where A_k is the k-th alternative project alternative, $V_k(A_k)$ is the aspected value for any variable A_k considered in the decisional process and w_k is the weight.

Finally, the preventive study of the failure rate was optimized to optimize a maintenance process and the mathematical resolution of a preventive maintenance problem was also proposed, in the presence of censored data.

The mathematical model can be effectively used where the sequence of life span and the sequences of the censored times are independent and where each of these sequences composed of independent random variables of the same order.

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