1. Introduction

Modern building structures consume too much energy and emit a number of hazardous pollutants, such as NOx, PM2.5, and CO2 into the atmosphere. Consequently, increasing the energy efficiency of buildings constitutes an important problem concerning building energy conservation.

Envelopes of modern buildings result in creation of major heating and cooling loads. Therefore, construction of an appropriate building envelope is an effective means of adjusting the heat-transfer rate and reducing the energy consumption of buildings. Thermal resistance and capacitance greatly impact the heat-transfer performance of building walls, and using these two parameters, one can analyze and optimize the heat-transfer performance of buildings, which in turn, would lead to construction of energy-saving building walls. Extant studies have
traditionally used only the thermal resistance and retaining-heat parameters to optimize the construction of building walls. Asan et al. [1,2] numerically explored the effect of different arrangements of wall materials on the time lag and decrement factor concerning the outdoor-air temperature as well as the relationship between these. Corresponding influencing factors were subsequently determined. Studies by del Coz Díaz et al. [3-4], Bouchair [5], and Li et al. [6] investigated optimization of the hollow-brick design based on the concept of equivalent thermal conductivity to realize high heat-insulation performance. However, thermal resistance and the retaining-heat parameter are not entirely independent in unsteady heat-transfer processes. Consequently, these cannot be used to analyze heat transfer and storage performances of building walls.

Recently, Guo et al. [7] proposed the concept of entransy—defined as half the product of internal energy and temperature \((G = \frac{1}{2} U \times T)\) \((U \text{ and } T\) denote the internal energy and temperature of a system). It has been observed that entransy dissipation occurs during heat-transfer processes owing to the existence of thermal resistance. Based on this result, thermal resistance can be defined as the ratio of the entransy-dissipation rate to square of the heat flow, and this definition of thermal resistance has been widely employed in the optimization of heat-transfer processes [8-18].

Based on the entransy method, the proposed study aims at decoupling the thermal resistance and capacitance with regard to unsteady heat-transfer processes, thereby subsequently illustrating weightiness of these two properties in heat-transfer processes and thence deducing the substitutional relationship between them.


For a general heat-conduction process occurring across a wall, the thermal energy conservation equation can be expressed as

\[
\rho c_p \frac{\partial T}{\partial t} = -\nabla \cdot q,
\]

where \(\rho\) and \(cp\) denote the density and constant-pressure volumetric specific heat of the wall, respectively; \(T\) denotes wall temperature; \(t\) represents time; and \(q\) denotes the heat-flow density. Multiplying both sides of Equation (1) by temperature \(T\) yields the balance equation of entransy during heat-transfer processes [19] expressed as

\[
\rho c_p T \frac{\partial T}{\partial t} = -\nabla \cdot (qT) + q \cdot VT,
\]

Terms on the left of the equality in Equation (2) denote the time variation of entransy stored per unit wall volume. Correspondingly, two terms exist on the right of the equality sign. The first denotes entransy transferred from one part of the wall to another while the second refers to the local entransy-dissipation rate during heat conduction. It can be seen from Equation (2) that entransy is dissipated as heat-transfer proceeds from a high-temperature zone of the wall to a low-temperature one.

Introducing the temperature of surrounding air into Equation (2), one gets

\[
\rho c_p T \frac{\partial T}{\partial t} = -\nabla \cdot (q(T - T_s)) - \nabla \cdot (qT_s) + q \cdot VT,
\]

where \(T_s\) denotes the temperature of surrounding air.

Integrating the entransy balance equation Equation (3) over the entire heat-conduction domain and transforming volume integrals to surface integrals over the domain boundary using the Gauss Law, Equation (4) can be obtained.

\[
\int (-q(T_s)) \cdot n dS - \int \rho c_p T \frac{\partial T}{\partial t} dV = \int \nabla T \cdot VT dV + \int (-q(T_s - T)) \cdot n dS,
\]

where \(n\) denotes the unit outward normal vector on the wall surface; \(\Sigma\) denotes surface integral area; \(\Omega\) represents volume integral area, and \(S\) and \(V\) represent the area and volume of the wall, respectively.

Based on Equation (4), heat-transfer impedance can be defined in accordance with the thermoelectricity analogy and described as follows by Equation (5).

\[
Z_a = \frac{T_s - T}{Q_b} = \frac{\int (-q(T_s)) \cdot n dS}{\int q \cdot n dS} - \frac{\int (qT_s) \cdot n dS}{\int q \cdot n dS}
\]

\[
\Phi_a = \int \rho c_p T \frac{\partial T}{\partial t} dV - \int \frac{\rho c_p T \frac{\partial T}{\partial t} dV}{\Phi_a} = \int (-q(T_s)) \cdot n dS \times \left[\int \rho c_p T \frac{\partial T}{\partial t} dV\right]
\]

\[
\Delta G_a = \int \rho c_p T \frac{\partial T}{\partial t} dV - \frac{\left[\int (-q(T_s)) \cdot n dS \times \int \rho c_p T \frac{\partial T}{\partial t} dV\right]}{q \cdot n dS},
\]
Here, subscripts 1 and 2 represent the two wall boundaries; \( Q_h \) represents the total heat flow rate; \( \Phi_h R \) denotes the enthalpy dissipation rate due to thermal resistance of the wall; and \( \Delta G_c \) represents the enthalpy variation rate owing to thermal capacitance.

Based on the above discussion, the thermal resistance \( R_h \) and capacitance \( X_h \) can be decoupled from the heat-transfer process and expressed as follows.

\[
R_h = \frac{(T_1 - T_2)}{\Phi_h} \tag{8}
\]

\[
X_h = \frac{(T_1 - T_2)^2}{\Delta G_c} \tag{9}
\]

The enthalpy-dissipation-based thermal resistance \( R_h \) and capacitance \( X_h \) are different from traditional thermal resistance \( R \) and retaining-heat parameter \( S \) in that the heat-transfer and heat-storage processes can be completely characterized by the thermal resistance \( R_h \) and capacitance \( X_h \), respectively.

3. Illustrative Example 1: Weightiness of Thermal Resistance and Capacitance in with regard to Building-wall Heat-transfer Processes

In accordance with the definition of thermal resistance \( R_h \) and capacitance \( X_h \), values of \( R_h \) and \( X_h \) can be easily calculated for a wall of a given room using Equations (8) and (9). Subsequently, the weightiness of thermal resistance and capacitance during the heat-transfer process can be determined.

3.1 Calculation model

A simplified two-plate room model \([20]\) was considered in this study. In the said model, the internal wall, floor, and ceiling were modeled as a single plate. An external wall, to where a south-facing window was fixed \([21]\) was modeled using another plate. The transient heat-conduction equation for the said plate modeled could be expressed as follows \([21]\).

\[
\rho \tau \frac{\partial T_p}{\partial t} = \frac{\partial}{\partial x} \left( k \frac{\partial T_p}{\partial x} \right) \tag{10}
\]

Boundary conditions for Equation (10) can be expressed as \([21]\).

\[
h_{\text{out}} (t_{\text{out},2} - t_{\text{in},1}) + q_{\text{e,}\text{out}} = -k_p \frac{\partial T_p}{\partial x} \bigg|_{x=0}, \tag{11}
\]

\[
h_{\text{in}} (t_{\text{in},2} - t_{\text{in},1}) + q_{\text{e,}\text{in}} = k_p \frac{\partial T_p}{\partial x} \bigg|_{x=L_p}, \tag{12}
\]

The initial condition for Equation (10) can be expressed as \([21]\).

\[
t_p(x, \tau) \bigg|_{\tau=0} = t_{p, \text{init}}, \tag{13}
\]

where \( \rho_p \), \( c_p \), and \( k_p \) denote the density, constant-pressure volumetric specific heat, and thermal conductivity of the plate, respectively; \( \tau \) refers to the plate temperature; \( \tau \) and \( x \) indicate time and space coordinates, respectively; \( h_{\text{out}} \) and \( h_{\text{in}} \) denote convective heat-transfer coefficients corresponding to the outer and inner plate surfaces, respectively, which could be calculated with reference to the ASHRAE Handbook \([22]\); \( t_{\text{out},a} \) and \( t_{\text{in},a} \) denote outdoor and indoor air temperatures, respectively, while \( t_{p,\text{out}} \) and \( t_{p,\text{in}} \) denote temperatures of the outer and inner plate surfaces, respectively; \( g_{t,\text{p,\text{out}}} \) and \( g_{t,\text{p,\text{in}}} \) represent thermal radiation heat fluxes corresponding to the outer and inner plate surfaces, respectively; \( L_p \) denotes plate thickness; and lastly, \( t_{\text{p,\text{init}}} \) denotes the initial plate temperature \([21]\).

Heat-transfer equations for the double-glazed window could be expressed as \([21]\).

\[
\rho_{\text{win},2} c_{\text{p,win},2} L_{\text{win},2} \frac{\partial T_{\text{win},2}}{\partial \tau} = h_{\text{out}} (t_{\text{out},2} - t_{\text{win},2}) + h_{\text{in}} (t_{\text{in},2} - t_{\text{win},2}) \tag{14}
\]

\[
+ h_{\text{2,2}} (t_{\text{win},1} - t_{\text{win},2}) + q_{\text{r,}\text{win},2},
\]

where \( \rho_{\text{win}} \) and \( c_{\text{p,win}} \) denote the density and constant-pressure volumetric specific heat of the window; subscripts 1 and 2 refer to the outer and inner glass layers, respectively; \( L_{\text{win}} \) and \( t_{\text{win}} \) denote the thickness of glass and its temperature, respectively; \( q_{\text{e,}\text{win}} \) denotes the thermal radiation heat flow density of the window; and lastly, \( h_{\text{2,2}} \) denotes the overall heat-transfer coefficient between the two glasses expressed as \([21]\).

\[
h_{\text{2,2}} = \frac{1}{U_{\text{win}}} \cdot \frac{1}{h_{\text{out}}} \cdot \frac{1}{h_{\text{in}}}, \tag{16}
\]

where \( U_{\text{win}} \) denotes the overall heat-transfer coefficient of the window while \( h_{\text{out}} ^* \) and \( h_{\text{in}} ^* \) refer to convective heat-transfer coefficients of the outer and inner window surfaces, respectively.

The energy-conservation equation is \([21]\).

\[
V_{\text{p}} \rho \tau \frac{\partial T_{\text{p,}\text{init}}}{\partial \tau} = \sum_{j=1}^{\infty} Q_{\text{p,j}} + Q_{\text{win}} + Q_{\text{d,c}} + Q_{\text{ven}}, \tag{17}
\]

where \( V_{\text{p}} \) denotes the room volume; \( \rho_p \) and \( c_{\text{p,win}} \) denote the density and constant-pressure volumetric specific heat of air; \( Q_{\text{p,j}} \), \( Q_{\text{win}} \), \( Q_{\text{d,c}} \), and \( Q_{\text{ven}} \) refer to convective
heat-transfer rates between the two plates and indoor air, window and indoor air, heat-transfer rate due to presence of internal heat sources, and that due to natural ventilation. Values of $Q_{p,j}$, $Q_{\text{win}}$ and $Q_{\text{ven}}$ could be obtained using following equations.

$$Q_{p,j} = h_{in} \times (t_{p,in,j} - t_{in,a}) \times A_{p,j},$$

$$Q_{\text{win}} = h_{in} \times (t_{\text{win,2}} - t_{in,a}) \times A_{\text{win}},$$

$$Q_{\text{ven}} = V_{R} \rho_{c} c_{p,a} \times \text{ACH} \times (t_{\text{out,a}} - t_{in,a}) / 3600,$$

wherein $A_p$ and $A_{\text{win}}$ denote areas of the plate and window while ACH represents air change per hour.

### 3.2 Calculation Conditions

Table 1 summarizes building parameters. In the following calculations, typical rooms in seven representative Chinese cities located in regions with different climatic conditions were considered (refer Table 2). Climate data for the seven cities were generated using the Chinese Architecture-specific Meteorological Data Sets for Thermal Environment Analysis.

### 3.3 Results Analysis

Calculation results for values of thermal resistance $R_h$ and capacitance $X_h$ of the external wall plate of the building in different cities are depicted in Figures 1–4.

### Table 1. Building parameters

<table>
<thead>
<tr>
<th>Category</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension</td>
<td>5.7 m (depth) × 3.6 m (width) × 3.2 m (height)</td>
</tr>
<tr>
<td>Wall and window</td>
<td>External reinforced concrete: 0.17 m; $\rho cp = 2.3$ MJ/m$^3$ K; $k = 2$ W/m K</td>
</tr>
<tr>
<td></td>
<td>External polystyrene board: 0.08 m; $\rho cp = 0.048$ MJ/m$^3$ K; $k = 0.047$ W/m K</td>
</tr>
<tr>
<td></td>
<td>External wall plate: 0.25 m; $\rho cp = 1.58$ MJ/m$^3$ K; $k = 0.14$ W/m K</td>
</tr>
<tr>
<td></td>
<td>Solar radiation absorptance of external wall plate: 0.6</td>
</tr>
<tr>
<td></td>
<td>Ceiling and floor: 0.2 m; $\rho cp = 2.3$ MJ/m$^3$ K; $k = 2$ W/m K</td>
</tr>
<tr>
<td></td>
<td>Internal wall: 0.2 m; $\rho cp = 0.84$ MJ/m$^3$ K; $k = 0.41$ W/m K</td>
</tr>
<tr>
<td></td>
<td>Internal wall plate: 0.2 m; $\rho cp = 1.5$ MJ/m$^3$ K; $k = 1$ W/m K</td>
</tr>
<tr>
<td></td>
<td>Double-glazing: 2.0 m (length) × 1.7 m (width)</td>
</tr>
<tr>
<td></td>
<td>Overall heat-transfer coefficient: 3.1 W/m$^2$ K</td>
</tr>
<tr>
<td></td>
<td>SC = 0.67 (winter), SC = 0.44 (summer)</td>
</tr>
<tr>
<td>ACH</td>
<td>5.0 h$^{-1}$ (outdoor temperature measured between 293 K and 299 K)</td>
</tr>
<tr>
<td></td>
<td>0.75 h$^{-1}$ (others)</td>
</tr>
<tr>
<td>Internal heat sources</td>
<td>10.6 W/m$^2$</td>
</tr>
<tr>
<td>Convective heat-transfer coefficient</td>
<td>$h_{\text{ae}} = 23.3$ W/m$^2$ K</td>
</tr>
<tr>
<td></td>
<td>$h_{\text{in}} = 1.31(\Delta t)^{0.22}$</td>
</tr>
</tbody>
</table>

### Table 2. Climatic characteristics of different cities considered in this study

<table>
<thead>
<tr>
<th>Regions</th>
<th>Latitude (o)</th>
<th>January</th>
<th>July</th>
<th>Climate Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Average</td>
<td>Average</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>temperature (°C)</td>
<td>solar radiation (W/m$^2$)</td>
<td>temperature (°C)</td>
</tr>
<tr>
<td>Harbin</td>
<td>45.75</td>
<td>-21.8</td>
<td>67.4</td>
<td>22.9</td>
</tr>
<tr>
<td>Urumchi</td>
<td>43.78</td>
<td>-18.2</td>
<td>56.8</td>
<td>25.0</td>
</tr>
<tr>
<td>Beijing</td>
<td>39.93</td>
<td>-3.6</td>
<td>97.8</td>
<td>25.3</td>
</tr>
<tr>
<td>Shanghai</td>
<td>31.17</td>
<td>3.7</td>
<td>86.7</td>
<td>29.2</td>
</tr>
<tr>
<td>Lhasa</td>
<td>29.67</td>
<td>-0.2</td>
<td>189.8</td>
<td>15.4</td>
</tr>
<tr>
<td>Kunming</td>
<td>25.02</td>
<td>7.5</td>
<td>168.1</td>
<td>19.5</td>
</tr>
<tr>
<td>Guangzhou</td>
<td>23.13</td>
<td>9.4</td>
<td>103.1</td>
<td>27.8</td>
</tr>
</tbody>
</table>
From the figures, it can be concluded that the weightiness of capacitance \( X_h \) exceeds that of thermal resistance \( R_h \) as regards the building external wall in summer. That is, heat storage plays a significant role in thermal processes that occur in buildings, and that optimization of the heat-storage process is more efficient compared to that of heat-transfer. Meanwhile, it was observed that the value of capacitance \( X_h \) was minimum in Harbin, whereas it was maximum in Lhasa.

Using the same method, values of the thermal resistance \( R_h \) and capacitance \( X_h \) during winter were calculated for the seven different cities. In this case, the weightiness of thermal resistance \( R_h \) was observed to exceed that of capacitance \( X_h \) for the building external wall, thereby implying that heat transfer plays a major role in the thermal process of buildings during winter, and that optimization of the heat-transfer process during winters is more efficient compared to that of the heat-storage process. As observed, the thermal resistance \( R_h \) was minimum in Kunming while that in Harbin was maximum.

Combining Equations (5)–(9), the following relationship between thermal resistance $R_h$ and capacitance $X_h$ can be deduced.

$$\frac{1}{Z_h} = \frac{1}{R_h} + \frac{1}{X_h}$$

(21)

It can, therefore, be demonstrated that thermal resistance $R_h$ and capacitance $X_h$ maintain an alternative relationship.

Multiplying both sides of Equation (21) by $(T_2 - T_1)$ and integrating the equation over the time domain yields:

$$Q = \int \frac{T_1 - T_2}{R_h} + \int \frac{T_1 - T_2}{X_h}$$

(22)

where $Q$ denotes the total heat transferred from outdoors to indoors in accordance with the building external wall plate.

For given values of $k$ and $\rho c_p$ of the building external wall plate, the value of $Q$ can be calculated using Equation (22). Likewise, for another value of $k$, one could calculate the value of $\rho c_p$ in accordance with Equation (22) under the condition that values $k$ and $Q$ are known.

The same room described in Table 1 was used to illustrate the said alternate relation. Figure 5 depicts the analysis result. As can be observed, the thermal conductivity $k$ and constant-pressure volumetric specific heat $\rho c_p$ of the building external wall plate in Beijing over an entire year demonstrates an approximate quadratic-function distribution.
5. Conclusions

Entransy-dissipation-based thermal resistance and capacitance of a building wall during unsteady heat transfer have been defined and decoupled in the proposed research. Accordingly, solutions to the following problems can be obtained.

(1) The weightiness of thermal resistance and capacitance during unsteady heat-transfer processes can be determined to guide the optimization of building wall structures. The heat-transfer and heat-storage processes can, accordingly, be pertinently optimized for transient heat-transfer problems.

(2) An alternate relationship between thermal resistance and capacitance can be established, and the same can be used to construct economical wall structures under the premise of ensuring optimum thermal performance of buildings.

(3) The proposed study provides a novel and in-depth view towards understanding unsteady heat-transfer processes, thereby addressing several problems related to building energy conservation.

Acknowledgements

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: the Youth Science Research Foundation of China Academy of Building Research (20160118331030053).

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DOI: https://doi.org/10.30564/jbms.v3i2.3103


