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# ARTICLE Non-fragile Dynamic Output Feedback H∞ Control for a Class of Uncertain Switched Systems with Time-varying Delay

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ARTICLE INFO	ABSTRACT
Article history Received: 7 May 2021 Accepted: 11 May 2021 Published Online: 13 May 2021	The problem of non-fragile dynamic output feedback $H_{\infty}$ control for a class of uncertain switched systems with time-varying delay is discussed. Firstly, the form of non-fragile dynamic output feedback $H_{\infty}$ controller is given. Under the condition that the upper bound of time delay and the upper bound of delay derivative are limited simultaneously, Lyapunov functional and its corresponding switching rules are constructed by using single Lyapunov function method and convex combination technique; Secondly, we use the inequality lemma to scale the derived Lyapunov functional in order to eliminate the time-varying delay term in the inequality, and then introduce the J-function to obtain a nonlinear matrix inequality that satisfies the $H_{\infty}$ performance index $\gamma$ , we also employ Schur complement lemma to transform the nonlinear matrix inequalities, a sufficient condition for the existence of a non-fragile dynamic output feedback $H_{\infty}$ controller and satisfying the $H_{\infty}$ performance index $\gamma$ is concluded for a class of uncertain switching systems with variable time delay; Finally, a switched system composed of two subsystems is considered and the effectiveness and practicability of the theorem are illustrated by numerical simulation with LMI toolbox.
Keywords:Single Lyapunov functionUncertainty $H_{\infty}$ controlNon-fragileOutput feedback	

### 1. Introduction

The switched system is a hybrid dynamical system consisting of a series of continuous-time subsystems and a rule that controls the switching between them. Liberzon D<sup>[1]</sup> summarized the research status of switching systems in the literature. The switched system has received extensive attention in the field of control. In the actual control a physical process, the uncertainty and time delay of the systems is often encountered, it is also the cause of systems instability and poor performance. In recent years, the problem of  $H_{\infty}$  control for linear time-delay systems with

uncertainties has attracted much attention. The research on uncertainty can be divided into two categories: norm bounded uncertainty and polymorphic uncertainty. For example, in 1992, Xie L<sup>[2]</sup> et al. studied the problem of robust H<sub> $\infty$ </sub> control for linear systems with norm-bounded time-varying uncertainties; In 2001, Goncalves E N<sup>[3]</sup> et al. studied the design of controllers for systems with polymorphic uncertainty. In terms of the design of the controller by the traditional robust control method only needs to find the gain of the controller to make the closed-loop system stable. However, when the controller is digitally executed, due to the limited word length of the memory and

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the aging of components and other factors, the controller is also very sensitive to small changes in its own when relying on different design algorithms, resulting in a closedloop system performance degradation or even unstable. It leads to a new problem: to design a controller that can withstand a certain range of parameter changes, that is, a non-fragile controller.

In 1997, Keel<sup>[4]</sup> and others first put forward the concept of "non-fragile". The explanation of fragile was further elaborated in some later published literature <sup>[5]</sup>. In recent years, more and more scholars are interested in non-fragile research. In 1998, Dorato P<sup>[6]</sup> and others studied the design of non-fragile controllers based on robust control. In addition, in many practical problems, the state of the systems can not be measured directly. In view of the cost and reliability of control implementation, it is more practical to adopt non fragile output feedback. Yu L<sup>[7]</sup> and others studied the design of robust memory less H<sub>a</sub> controllers for linear time-delay systems with time-varying and bounded norms uncertainties; In 1997, Han H C<sup>[8]</sup> and others studied the LMI method of output feedback H<sub>a</sub> controller design for linear time-delay systems; In 1998, Kokame H<sup>[9]</sup> and others studied the robust H<sub>m</sub> performance of linear time-delay differential systems with time-varying uncertainties; In 2006, Yang G H<sup>[10]</sup> and others studied the design of non-fragile output feedback H<sub>w</sub> controller for linear systems; In 2009, Li L<sup>[11]</sup> and others studied the non-fragile dynamic output feedback control of linear time-varying delay systems; In 2010, Y Wang<sup>[12]</sup> and others studied the design of non-fragile output feedback H<sub>∞</sub> controllers for linear systems; In 2013, Fernando T L<sup>[13]</sup> and others studied the output feedback guaranteed cost control problem of uncertain linear discrete systems. However, there are few researches on the non-fragile dynamic output feedback  $H_{\infty}$  control of time-varying time-delay switched systems with uncertainties.

In this paper, the problem of dynamic output feedback  $H_{\infty}$  control for a class of uncertain time-varying delay systems is studied. Assuming that the uncertainty is norm bounded and parameter uncertainty which the parametric form of the uncertain coefficient matrix is given. The dynamic output feedback controller is designed, which is substituted into the original systems to obtain a new closed-loop system. The corresponding switching rules and Lyapunov functional are constructed. By using convex combination technique, inequality scaling lemma, and Schur complement lemma, the set of linear matrix inequality satisfying the given  $H_{\infty}$  performance index  $\gamma$  is obtained and the original system is asymptotically stable and satisfies the performance index  $\gamma$ . The sufficient conditions for  $H_{\infty}$  control are obtained, and the validity of the

theorem is verified by numerical simulation.

#### 2. Description of the Problem

Consider a class of switched systems with time delay and uncertainties.

$$\dot{x}(t) = (A_{\sigma(t)} + \Delta A_{\sigma(t)}(t))x(t)$$
$$+ (A_{d\sigma(t)} + \Delta A_{d\sigma(t)}(t))x(t - d(t))$$
$$+ B_{\sigma(t)}u(t) + B_{1\sigma(t)}\omega(t)$$

$$z(t) = C_{1\sigma(t)}x(t) + D_{1\sigma(t)}\omega(t)$$

$$y = C_{2\sigma(t)} x(t) + D_{2\sigma(t)} \omega(t)$$

$$x(t) = 0, t \in [-h, 0)$$
 (1)

Where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^r$ ,  $z(t) \in \mathbb{R}^m$ ,  $y(t) \in \mathbb{R}^g$  and  $\omega(t) \in \mathbb{R}^q$  are state vector, control input vector, control output vector, measurement output vector and external disturbance input vector.  $A_i$ ,  $A_{di}$ ,  $B_i$ ,  $B_{1i}$ ,  $C_{1i}$ ,  $C_{2i}$ ,  $D_{1i}$ ,  $D_{2i}$  are constant matrices with proper dimensions; d(t) is a time-varying delay and is a continuous function satisfying:  $0 \le d(t) \le \tau$ ,  $\dot{d}(t) \le d < 1$ .  $\tau$ , d are known constants.  $\omega(t) \in \mathbb{R}^m$  is the interference input of limited energy;  $\sigma : [0, \infty) \rightarrow M = \{1, 2, \cdots, n\}$  represents the handover rule of this systems, where *n* represents the *n* switching subsystems of the switching system.  $\Delta A_i(t)$  and  $\Delta A_{di}(t)$  represent the time-varying uncertainty in the model and satisfy the following form of norm bounded conditions

$$\Delta A_i(t) = H_{1i}F_i(t)E_{1i} \tag{2}$$

$$\Delta A_{di}(t) = H_{2i}F_i(t)E_{2i} \tag{3}$$

Where  $H_{1i}$ ,  $H_{2i}$ ,  $E_{1i}$  and  $E_{2i}$  are constant matrices with appropriate dimensions,  $F_i(t)$  are unknown matrices with Lebesgue measurable elements and satisfy the conditions

$$F_i^T(t)F_i(t) \le I \tag{4}$$

The problem studied in this paper is to design dynamic non-fragile output feedback  $H_{\infty}$  controller for uncertain time-varying delay systems (1), as follows:

$$\dot{x}_{c}(t) = (A_{ci} + \Delta A_{ci}(t))x_{c}(t) + (B_{ci} + \Delta B_{ci}(t))y(t)$$

$$u_i(t) = (C_{ci} + \Delta C_{ci}(t))x_c(t)$$
 (5)

makes the systems (1) be asymptotically stable and meet the given performance index  $\gamma$ . The uncertain terms  $\Delta A_{ci}(t)$ ,  $\Delta B_{ci}(t)$  and  $\Delta C_{ci}(t)$  of the controller (5) represent uncertain gain perturbation and meet the conditions:

$$\Delta A_{ci}(t) = H_{3i}F_i(t)E_{3i}$$
$$\Delta B_{ci}(t) = H_{4i}F_i(t)E_{4i}$$

$$\Delta C_{ci}(t) = H_{5i} F_i(t) E_{5i} \tag{6}$$

condition (4) and (6), where  $H_{3i}$ ,  $H_{4i}$ ,  $H_{5i}$ ,  $E_{3i}$ ,  $E_{4i}$  and  $E_{5i}$  are constant matrices with appropriate dimensions. And (5) where  $A_{ci}$ ,  $B_{ci}$  and  $C_{ci}$  are undetermined constant matrices with appropriate dimensions.

The closed-loop system is obtained by combining formula (1) and formula (5)

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_{c}(t) \end{bmatrix} = \begin{bmatrix} A_{\sigma(t)} + \Delta A_{\sigma(t)}(t) & B_{\sigma(t)}(C_{ci} + \Delta C_{ci}(t)) \\ (B_{ci} + \Delta B_{ci}(t))C_{2\sigma(t)} & A_{ci} + \Delta A_{ci}(t) \end{bmatrix}$$

$$* \begin{bmatrix} x(t) \\ x_{c}(t) \end{bmatrix} + \begin{bmatrix} A_{d\sigma(t)} + \Delta A_{d\sigma(t)}(t) \\ 0 \end{bmatrix} * x(t - d(t)$$

$$+ \begin{bmatrix} B_{1\sigma(t)} \\ (B_{ci} + \Delta B_{ci}(t))D_{2\sigma}(t) \end{bmatrix} * \omega(t)$$
(7)

Aiming at the systems in this case, the problem to be solved in this paper is to find a sufficient condition for the existence of a non-fragile output feedback controller that satisfies the given  $H_{\infty}$  performance index  $\gamma$  for the uncertain time-varying delay system (1).

#### 3. Main Results

Definition 1 Considering any given constant  $\gamma$ , uncertain time-varying time-delay switched system (1) in the presence of non-fragile dynamic output feedback gain with the form of additive perturbation, the Existing form of non-fragile dynamic output feedback  $H_{\infty}$  controller can be obtained, and the system satisfies the given constant  $\gamma$ . By using the corresponding switching rules, the systems (1) can be stabilized if there is an output feedback  $H_{\infty}$  controller of the form (5) and corresponding switching rules  $\sigma(t)$ . For a given performance index, the response of the closed-loop systems satisfy the following two conditions:

1) When external disturbance  $\omega(t) = 0$ , construct corresponding switching rules  $\sigma(t)$ , so that the systems (1) are asymptotically stable.

2) When the initial state of system (1) at t=0 is 0, the following inequality holds for all non-zero values  $(t) \in L[0,T], 0 \le T < \infty$ , as follows:

$$\left\|z\right\|_{2} \le \gamma \left\|\omega(t)\right\|_{2}$$

Lemma 1 <sup>[12]</sup> For any constant  $\varepsilon > 0$ , vector X, Y, there are

$$2X^T Y \le \varepsilon X^T X + \varepsilon^{-1} Y^T Y$$

Lemma  $2^{[12]}$  For any constant  $\varepsilon > 0$  and proper dimensions matrices *H*, *E* and *F*(*t*) satisfy  $F^{T}(t)F(t) \le I$ , there are

$$HF(t)E + E^{T}F^{T}(t)H^{T} \leq \varepsilon^{-1}HH^{T} + \varepsilon E^{T}E$$

Theorem 1 For systems (1), the dynamic output feedback control law given by equation (5), where the controller parameter matrices are as follows:

$$A_{ci} = r_{1i}P_c^{-1}$$

$$B_{ci} = r_{2i}P_c^{-1}C_{2i}^{T}$$

$$C_{ci} = r_{3i}B_i^{T}P$$
(8)

Given a constant  $\gamma > 0$ , there are the following linear matrix inequalities

$$\sum_{i=1}^{n} \lambda_{i} \begin{bmatrix} M_{1i} & M_{2i} \\ M_{2i}^{T} & M_{3i} \end{bmatrix} < 0, \sum_{i=1}^{n} \lambda_{i} \begin{bmatrix} N_{1i} & N_{2i} \\ N_{2i}^{T} & N_{3i} \end{bmatrix} < 0$$
(9)

having solutions for P > 0,  $P_c > 0$ ,  $r_{1i}$ ,  $r_{2i}$ ,  $r_{3i}$ , given  $\mathcal{E}_{\alpha i} > 0$ , (i( $\alpha = 1 \ 2 \ \cdots \ 7$ ),  $\lambda_i \ge 0$  ( $i \in \overline{N}$ ) and satisfying  $\sum_{i=1}^n \lambda_i = 1$  of n real number, then the systems (1) are asymptotically stable under the action of controller (5) and satisfy the given  $H_c$  norm bound  $\gamma$ . Where

$$M_{1i} = A_{i}^{T}P + PA_{i} + C_{1i}^{T}C_{1i} + C_{2i}^{T}C_{2i} + \varepsilon_{1i}E_{1i}^{T}E_{1i}$$
$$+\varepsilon_{3i}E_{2i}^{T}E_{2i} + \varepsilon_{7i}C_{2i}^{T}E_{4i}^{T}E_{4i}C_{2i} + \varepsilon_{2i}I$$
$$M_{2i} = \left[C_{1i}^{T}D_{1i} + PB_{1i}PH_{1i}PH_{2i}PA_{di}PB_{i}H_{5i}PB_{i}\right]$$

$$M_{3i} = diag \begin{bmatrix} D_{1i}^{T} D_{1i} + D_{2i}^{T} D_{2i} + \varepsilon_{4i} D_{2i}^{T} E_{4i}^{T} E_{4i} D_{2i} \\ -\gamma^{2} I - \varepsilon_{1i} I - [(1-d)\varepsilon_{3i}] I \\ -[(1-d)\varepsilon_{2i}] I] - \varepsilon_{5i} I - r_{3i}^{-2} I \end{bmatrix}$$
$$N_{1i} = 2r_{1i}I + 2r_{2i}^{2}C_{2i}^{T}C_{2i} + \varepsilon_{6i}E_{3i}^{T}E_{3i} + \varepsilon_{5i}E_{5i}^{T}E_{5i}$$

$$N_{2i} = \begin{bmatrix} P_c H_{3i} & P_c H_{4i} & P_c H_{4i} & PB_i \end{bmatrix}$$

$$N_{3i} = diag \begin{bmatrix} -\varepsilon_{6i}I & -\varepsilon_{7i}I & -\varepsilon_{4i}I & -I \end{bmatrix}$$

The switching rules are as follows:

$$\sigma(t) = \arg\min\left\{\begin{bmatrix} x(t) \\ \omega_i(t) \end{bmatrix}^{\mathrm{T}} R'_i \begin{bmatrix} x(t) \\ \omega_i(t) \end{bmatrix} + x_c^T(t) S'_i x_c(t), i \in M \right\}$$
(10)

Where

1

$$R_{i}^{\prime} = \begin{bmatrix} R_{i} + C_{1i}^{T}C_{1i} & C_{1i}^{T}D_{1i} + PB_{1i} \\ D_{1i}^{T}C_{1i} + B_{1i}^{T}P & D_{1i}^{T}D_{1i} - \gamma^{2}I + D_{2i}^{T}D_{2i} \\ + \varepsilon_{4i}D_{2i}^{T}E_{4i}^{T}E_{4i}D_{2i} \end{bmatrix}$$

$$S'_{i} = S_{i} + r_{2i}^{2}C_{2i}^{T}C_{2i} + \varepsilon_{4i}^{-1}P_{c}H_{4i}H_{4i}^{T}P_{c}$$

$$\begin{split} R_{i} &= A_{i}^{T}P + PA_{i} + \varepsilon_{1i}E_{1i}^{T}E_{1i} + \varepsilon_{2i}I \\ &+ \varepsilon_{3i}E_{2i}^{T}E_{2i} + C_{2i}^{T}C_{2i} + \varepsilon_{7i}C_{2i}^{T}E_{4i}^{T}E_{4i}C_{2i} \\ &+ P[\varepsilon_{1i}^{-1}H_{1i}H_{1i}^{T} + [(1-d)\varepsilon_{2i}]^{-1}A_{di}A_{di}^{T} \\ &+ [(1-d)\varepsilon_{3i}]^{-1}H_{2i}H_{2i}^{T} + r_{3i}^{2}B_{i}B_{i}^{T} \\ &+ \varepsilon_{5i}^{-1}B_{i}H_{5i}H_{5i}^{T}B_{i}]P \\ S_{i} &= A_{ci}^{T}P_{c} + P_{c}A_{ci} + \varepsilon_{6i}E_{3i}^{T}E_{3i} + \varepsilon_{5i}E_{5i}^{T}E_{5i} \\ &+ r_{2i}^{2}C_{2i}^{T}C_{2i} + P_{c}(\varepsilon_{6i}^{-1}H_{3i}H_{3i}^{T} \\ &+ \varepsilon_{7i}^{-1}H_{4i}H_{4i}^{T})P_{c} + PB_{i}B_{i}^{T}P \end{split}$$

When the switched systems (1) have the non-fragile dynamic output feedback  $H_{\infty}$  controller, the uncertain switched systems with time-varying delay (1) are asymptotically stable and satisfy the given performance index  $\gamma$ .

Proof For systems (6), let

$$\{(t_k, i_k) \mid i \in M; k = 0, 1, 2, \dots, 0 = t_0 \le t_1 \le \dots\}$$

be the switching sequence formed on  $t \in [0, \infty)$  corresponding to the switching rule. Therefore, the Lyapunov function has the following form:

$$V(t) = V_1(t) + V_2(t)$$
$$V_1(t) = \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & P_c \end{bmatrix} \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix}$$

$$V_2(t) = \int_{t-d(t)}^t x^T(s) (\varepsilon_{2i}I + \varepsilon_{3i}E_{2i}^T E_{2i}) x(s) ds$$

The time derivative along the systems (6) is obtained, as follows:

$$\dot{V}(t) = \dot{x}^{T}(t)Px(t) + x^{T}(t)P\dot{x}(t) + \dot{x}_{c}^{T}(t)P_{c}x_{c}(t) + x_{c}^{T}(t)P_{c}\dot{x}_{c}(t) + x^{T}(t)(\varepsilon_{2i}I + \varepsilon_{3i}E_{2i}^{T}E_{2i})x(t) - (1 - \dot{d}(t))x^{T}(t - d(t))(\varepsilon_{2i}I + \varepsilon_{3i}E_{2i}^{T}E_{2i})x(t - d(t))$$

Firstly, the internal stability of the systems is considered, set  $\omega(t) = 0$ , combine lemma 1, 2 and equations

(2), (3), (4), (5), (6), there are

$$\dot{V}(t) \le x^{T}(t)R_{i}x(t) + x_{c}^{T}(t)S_{i}x_{c}(t)$$
 (11)

According to equation (11), as long as  $R_i < 0$ ,  $S_i < 0$ , there is  $\dot{V}(t) < 0$ . Then the systems (7) is asymptotically stable.

Secondly, when  $\omega(t) \neq 0$ , under zero initial conditions  $x(t_0) = 0$ , the J function is introduced,

$$J = \int_0^\infty (Z_i^T Z_i(t) - \gamma^2 \omega_i^T(t) \omega(t)) dt \le$$

$$\int_0^\infty (Z_i^T Z_i(t) - \gamma^2 \omega_i^T(t) \omega(t) + \dot{V}(t)) dt \le$$
$$\int_0^\infty \left[ x^T(t) \quad \omega_i^T(t) \right] R_i' \left[ \begin{array}{c} x(t) \\ \omega_i(t) \end{array} \right] dt + \int_0^\infty x_c^T(t) S_i' x_c(t) dt$$

where

$$R_{i}^{\prime} = \begin{bmatrix} R_{i} + C_{1i}^{T}C_{1i} & C_{1i}^{T}D_{1i} + PB_{1i} \\ D_{1i}^{T}C_{1i} + B_{1i}^{T}P & D_{1i}^{T}D_{1i} - \gamma^{2}I + D_{2i}^{T}D_{2i} \\ + \varepsilon_{4i}D_{2i}^{T}E_{4i}^{T}E_{4i}D_{2i} \end{bmatrix}$$

$$S'_{i} = S_{i} + r_{2i}^{2} C_{2i}^{T} C_{2i} + \varepsilon_{4i}^{-1} P_{c} H_{4i} H_{4i}^{T} P_{c}$$

When  $R'_i < 0$ ,  $S'_i < 0$ , there are J < 0. The systems (1) satisfy the H<sub>∞</sub> performance index  $\gamma$  and are gradually stable under the action of the non-fragile dynamic output feedback controller (4). The Schur complement lemma is further used to make the  $R'_i < 0$ ,  $S'_i < 0$ . It is equivalent to LMI of formula (8) in the theorem. And in the case of no disturbance input,  $\dot{V}(t) < 0$  can still hold, then the theorem is proved.

#### 4. Simulation Example

Given two uncertain time-varying delay continuous subsystems to form a linear switched systems (1). Where

$$A_{1} = \begin{bmatrix} -5 & -2 \\ 0 & -6 \end{bmatrix}, A_{2} = \begin{bmatrix} -5.5 & 1 \\ -1 & -6 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} 0 & 0.3 \\ 0.1 & 0.4 \end{bmatrix}, A_{d2} = \begin{bmatrix} 0 & 0.3 \\ -0.2 & 0.5 \end{bmatrix},$$
$$B_{1} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, B_{2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, B_{11} = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}, B_{12} = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix},$$
$$C_{11} = C_{12} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, C_{21} = C_{22} = \begin{bmatrix} 1 & 3 \end{bmatrix},$$
$$D_{11} = D_{12} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, D_{21} = D_{22} = 1,$$
$$H_{11} = H_{12} = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, E_{11} = E_{12} = \begin{bmatrix} 0.2 & 0 \end{bmatrix},$$
$$E_{21} = E_{22} = \begin{bmatrix} 0.5 & 0 \end{bmatrix}, E_{41} = E_{42} = 0.1,$$
$$H_{51} = H_{52} = 0.1,$$
$$H_{21} = H_{22} = \begin{bmatrix} 0.3 \\ 0.15 \end{bmatrix},$$
$$H_{31} = H_{32} = H_{41} = H_{42} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix},$$
$$F_{1}(t) = F_{2}(t) = \sin(t).$$

The time delay is taken as 0.5,  $\lambda_1 = 0.4$ ,  $\lambda_2 = 0.6$ , d = 0.9,  $\mathcal{E}_{\alpha i} = 1$ , ( $\alpha = 1 \ 2 \ \cdots \ 7$ ; i = 1, 2. By solving linear matrix inequality (9) to get

$$P = \begin{bmatrix} 123.8791 & -83.3075 \\ -83.3075 & 57.8461 \end{bmatrix},$$
$$P_{c} = 1 \times 10^{8} \begin{bmatrix} 1.8904 & -0.9448 \\ -0.9448 & 0.4728 \end{bmatrix}$$

We substitute  $r_{11} = -2.8628 \times 10^8$ ,  $r_{21} = 5.7130 \times 10^3$ ,  $r_{31} = 7.2021 \times 10^{-5}$  into (8) to get

$$A_{c1} = 1 \times 10^{3} \begin{bmatrix} -1.3283 & -2.6546 \\ -2.6546 & -5.3112 \end{bmatrix}$$

$$B_{c1} = \begin{bmatrix} 0.1854\\ 0.3709 \end{bmatrix}, C_{c1} = \begin{bmatrix} 0.0059 & -0.0039 \end{bmatrix};$$

We substitute  $r_{12} = -4.2942 \times 10^8$ ,  $r_{22} = 4.7629 \times 10^3$ ,  $r_{32} = 7.2021 \times 10^{-5}$  into (8) to get

$$A_{c2} = 1 \times 10^{3} \begin{bmatrix} -1.9924 & -3.9819 \\ -3.9819 & -7.9668 \end{bmatrix},$$
$$B_{c2} = \begin{bmatrix} 0.1546 \\ 0.3093 \end{bmatrix}, C_{c2} = \begin{bmatrix} -0.0060 & 0.0042 \end{bmatrix}$$

The switching rules are as follows:

$$\sigma(x(t)) = i = \begin{cases} x(t) \\ \omega_i(t) \end{bmatrix}^{T} (R'_1 - R'_2) \begin{bmatrix} x(t) \\ \omega_i(t) \end{bmatrix} \\ 1.if + x_c^T(t)(S'_1 - S'_2)x_c(t) < 0 \\ 2.if \begin{bmatrix} x(t) \\ \omega_i(t) \end{bmatrix}^{T} (R'_2 - R'_1) \begin{bmatrix} x(t) \\ \omega_i(t) \end{bmatrix} \\ + x_c^T(t)(S'_2 - S'_1)x_c(t) < 0 \end{cases}$$

#### 5. Conclusions

In this paper, we study the problem of non-fragile dynamic output feedback H<sub>a</sub> control for a class of switched systems with time-varying delay and norm-bounded uncertainty, a suitable switching rule is constructed by using a convex combination method and single Lyapunov function method. Furthermore, the derivative term of Lyapunov functional is scaled by using inequality lemma, and a sufficient condition for the problem to be solved is obtained in the form of matrix inequality which is only related to the upper bound of delay derivative. By using Schur complement lemma, the obtained matrix inequality is decomposed into a set of linear matrix inequalities. At the same time, we get the sufficient conditions of non-fragile H<sub>op</sub> control for the uncertain switched systems with time-varying delay and present the specific form of the non-fragile dynamic output feedback  $H_{\infty}$  controller. The practicality and validity of the theorem are verified by numerical simulation examples. The results show that the non-fragile dynamic output feedback H<sub>∞</sub> controller theorem is valid for the systems, and the design method may be suitable for other types of systems.

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